

City Size and the Demand for Local Public Goods[†]

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Abstract: This paper studies how size-induced cost differences in the provision of local public goods affect the efficient level of public spending. Since public goods are non-rival in consumption, the per-capita cost of a given level of public good provision is lower in more populous jurisdictions. We show that this cost advantage gives rise to a substitution of public for private consumption and specify conditions under which the efficient level of local public expenditures per capita rises with a jurisdiction's population size.

Keywords: Local Public Goods; Congestion; Population Size; Fiscal Need

JEL Classification: H72, R51, H73

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1 Introduction

It has long been recognized in the public finance literature that local public spending per-capita tends to show a positive association with city size (*e.g.*, Popitz, 1932, Schmandt and Stephens, 1963). Also today, this pattern is a stylized fact of local government finances. In the US, for instance, the Census of Governments documents a monotonous increase in average per capita spending of municipalities across different size classes (see Table ??).

To explain this pattern, several strands of the literature point at urban disadvantages that tend to raise the cost of public service provision in larger cities or municipalities. These disadvantages include, *inter alia*, the higher prevalence of certain social problems such as crime (*e.g.*, Glaeser and Sacerdote, 1999), poverty and income inequality (Alesina, Baqir, and Easterly, 2000, Borge and Rattsø, 2004) as well as benefit spillovers to non-residents (*e.g.*, Chernick and Reschovsky, 1997). Other explanations emphasize the role of the bureaucracy (Nellor, 1984) and various political economy considerations (*e.g.*, Alesina and Glaeser, 1995, Baqir, 2002).

All these explanations are based on the notion that providing a given level or quality of public services is more costly in larger cities. Yet, this view is at odds with the traditional literature on urban public finance which has generally argued that the cost of public service provision in per-capita terms would decline with city size due to non-rivalry in the consumption of public goods.¹

This paper argues that, rather than reflecting disadvantages in *supplying* public services, the posi-

¹Following the seminal papers by Borcherting and Deacon (1972) and Bergstrom and Goodman (1973) much research has investigated empirically the effects of population size and density on the cost of public service provision. While the empirical estimates in this early literature point at a high degree of rivalry in the consumption of public goods, this conclusion has been challenged on methodological grounds (*e.g.*, Oates, 1988, Edwards, 1990, Means and Mehay, 1995, Reiter and Weichenrieder, 1997). Studies focusing on specific functions of government that use indicators for the quality of public service provision, confirm substantial cost advantages for larger jurisdictions (*e.g.*, McMillan, Wilson, and Arthur, 1981, Brueckner, 1982, Craig, 1987).

Table 1: General expenditures of US municipalities (USD per capita)

Population Size Group (in 1000)	> 300	≥200 <300	≥100 <200	≥75 <100
	2,884	1,587	1,463	1,326
Population Size Group (in 1000)	≥50 <75	≥25 <50	≥10 <25	≥75 <10
	1,254	1,151	1,085	891

Own computations based on U.S. Census Bureau (2005: p.27). General expenditures refer to 2001-2002, population size refers to 2000.

tive association between per-capita government spending and population size observed in the data could also be explained by a higher *demand* for public consumption. Since public goods are non-rival, the per-capita cost of a given level of public goods provision is lower in more populous jurisdictions. Based on a standard model of local public goods (for a survey see Wildasin, 1987), we show that, in an efficient allocation, this cost advantage gives rise to a substitution from private to public consumption. Under certain conditions, the efficient allocation of resources would also require public spending in larger cities to exceed that in smaller jurisdictions even in per-capita terms.

Our analysis opens up a new perspective on the “fiscal need” or “expenditure need” of cities, concepts used in several countries to determine the allocation of unconditional grants among local governments. For example in Spain, Germany, and Austria, larger jurisdictions are assumed to have a higher fiscal need in per-capita terms which positively affects the per capita level of unconditional grants obtained from higher layers of government.² Similarly, Australia, the United Kingdom, or

²In these countries the allocation of unconditional grants is based on notional or weighted rather than actual

France use indicators of population density, urbanization, or population size in the determination of expenditure needs (see Reschovsky, 2007). These grant allocation rules have been criticized as a subsidization of cities and their inefficient public sectors (*e.g.*, Fenge and Meier, 2002). However, our analysis suggests that the favorable treatment of larger cities could possibly be justified on efficiency grounds, since the demand for public services increases with population size.

The paper is organized as follows. The next section develops the basic theoretical model. It describes the efficient allocation in the presence of interjurisdictional productivity differentials which give rise to population size differences between jurisdictions and discusses how the efficient allocation of resources between private and public consumption is affected by those differences. Section 3 elaborates on the implications for public spending per-capita and shows under which conditions smaller jurisdictions will specialize in the consumption of private goods. Section 4 concludes.

2 Efficiency in a Model with Local Public Goods

Let us consider an economy with two jurisdictions, $i = 1, 2$. The population consists of $n > 2$ households. A number n_i of households, each of which is endowed with one unit of labor, are located in jurisdiction i . Of the total land area (t) a fraction (t_i) is assigned to jurisdiction i , the population numbers. Formally, fiscal need fn_i is defined as

$$fn_i = z n_i w_i,$$

where z is a basic indicator of fiscal need per capita, n_i is the number of inhabitants, and w_i is a weight or factor that increases with a jurisdiction's population size. For instance, in Austria the weights for municipalities are set at 1.6 if the population is below 10,000 inhabitants and are increasing with population size up to a figure of $2\frac{1}{3}$ for municipalities with more than 50,000 inhabitants (Genser, 2001, and Schratzenstaller, 2008). In Spain, the weight is unity for the calculation of fiscal need of jurisdictions with less than 5,000 inhabitants, and rises to 2.85 for cities with more than 500,000 inhabitants (Solé-Ollé and Bosch, 2005). In Germany, different rules apply across states. For example, the most populous state, North Rhine-Westfalia, displays weights that vary between unity for municipalities below 25,000 inhabitants and a figure of 1.51 for cities with more than 635,500 inhabitants (Buettner and Holm-Hadulla, 2008, and Gemeindefinanzierungsgesetz Nordrhein-Westfalen, 2011).

remaining part is used by the other jurisdiction. The land available to jurisdiction i is combined with labor to produce a uniform consumption good (x_i) according to a production function, which is identical across jurisdictions except for a possible difference in total factor productivity. Allowing for such a difference proves important in the analysis below since it gives rise to a non-trivial population size distribution of jurisdictions.

Households derive utility from the consumption of the private and a public good (z_i) with preferences being described by the strictly quasi-concave utility function $u_i = u(x_i, z_i)$.³ We assume that the public good is produced at cost $C(z_i, n_i)$ which is increasing in both the level or quality of the public service provided and the number of households located in the jurisdiction. To characterize an efficient allocation of resources in this economy, the literature on urban public finance has considered the decision problem of a “benevolent” central planner aiming to maximize the utility of a representative household under the constraint that utility is the same in all possible locations (*e.g.*, Wildasin, 1987). A somewhat simpler approach is to consider this decision problem under the premise that the central planner aims at maximizing the level of utility of all residents regardless of their location. The objective of the central planner is to maximize the residents’ utility subject to the general resource constraint.

$$\mathcal{L} \equiv u + \lambda [\gamma F(n_1, t_1) + F(n - n_1, t - t_1) - x(z_1, u)n_1 - x(z_2, u)(n - n_1) - C(z_1, n_1) - C(z_2, n - n_1)], \quad (1)$$

where n_2, t_2 are replaced by $n - n_1, t - t_1$ and $x(z_i, u)$ gives the amount of private goods required

³Since our focus is on the choice between public and private consumption, scarcity of land is treated here simply as a constraint to production. An alternative way to discuss land use is to follow the urban economics literature, introducing demand for land by consumers. This would allow us to discuss related substitution effects. However, exploration is left for future research.

to sustain a given level of utility u at a level of public goods provision z_i . $\gamma \geq 1$ shifts the total factor productivity in jurisdiction 1. This objective function allows us to derive four first-order conditions:

$$\lambda[\gamma F_{1n} - x_1 - C_{1n}] - \lambda[F_{2n} - x_2 - C_{2n}] = 0 \quad (2)$$

$$-\lambda \left[n_1 \frac{\partial x_1}{\partial z_1} + C_{1z} \right] = 0 \quad \text{and} \quad -\lambda \left[(n - n_1) \frac{\partial x_2}{\partial z_2} + C_{2z} \right] = 0 \quad (3)$$

$$\lambda \gamma F_{1t} - \lambda F_{2t} = 0 \quad (4)$$

The first subscript refers to the region, the second subscripts z , n and t denote the partial derivative with respect to the public good, the local population and land, respectively. Equation (??) is the so-called locational efficiency condition, requiring that the marginal product of labor net of the costs for providing the marginal household with consumption goods is equalized across jurisdictions. The optimal supply of public services is ensured by equations (??), which correspond to the well known Samuelson condition according to which the sum of the marginal rates of substitution is equal to the location specific marginal rate of transformation between private and public consumption. Equation (??) characterizes the optimal allocation of land among the two jurisdictions. We can use the four optimality conditions (??) - (??) to determine the allocation of population n_1 , land t_1 , and the level of public good provision in each jurisdiction z_1, z_2 .

With differences in productivity, the efficient allocation is characterized by different population sizes. A comparative static exercise allows us to state the following relationship between the productivity parameter and the population size:

Lemma 1 (Population Size of Jurisdictions)

In an efficient allocation, where the level of utility is uniform across jurisdictions, the jurisdiction with the higher total productivity parameter γ , ceteris paribus, has a larger population size.

The proof is given in Appendix ??.⁴

Our analysis assumes that the total land endowment is given – an increase of the land-size of one jurisdiction implies a land-size reduction for the other. The literature on local public finance has also dealt with a setting where each jurisdiction has fixed land size t but where the number of jurisdictions is subject to consideration by the central planner who has access to land at some fixed rental rate ρ , which may reflect opportunity cost of land (see Wildasin, 1987). In such a setting, land use is efficient when the number of jurisdictions is chosen to maximize utility. Suppose all jurisdictions have standard level of productivity except for jurisdiction 1 and let m_2 be the number of these type-2 jurisdictions.⁵ Following Wildasin (1987) by assuming that the land must be paid for from the production of the numeraire good, we can specify the modified objective function as:

$$\mathcal{L} \equiv u + \lambda \left[\gamma F(n_1, t) + m_2 F\left(\frac{n - n_1}{m_2}, t\right) - x(z_1, u) n_1 - x(z_2, u) (n - n_1) - C(z_1, n_1) - m_2 C\left(z_2, \frac{n - n_1}{m_2}\right) - \rho t (1 + m_2) \right],$$

⁴Since Lemma 1 focuses on total factor productivity, it might be interesting to discuss briefly whether the result is robust if the productivity increase is biased in favor of one of the input factors. Consider the case where the productivity increase is quasi *land-augmenting* with region 1's output being $F(n_1, \gamma t_1)$. Since the cross-partial derivative of the production function with respect to the input factors is positive, this would further increase the marginal productivity of labor in the region experiencing the productivity shock. Therefore, Lemma 1 continues to hold under this condition. If the productivity increase is *labor-augmenting* with region 1's output being $F(\gamma n_1, t_1)$, however, the effect on the marginal product of labor is ambiguous. Depending on the production function, in particular, on the second partial derivatives, a decline in the marginal product of labor may result, and jurisdictions with low productivity may be characterized by a larger population size. However, the subsequent analysis of the relationship between the efficient level of public service provision and the population size would still apply.

⁵If the central planner is free to choose whether to add low or high productivity cities, it is obvious that we end up in a setting with only one type of city and have a trivial population size distribution.

where we make use of the fact that all type-2 jurisdictions are fully symmetric and, thus, all have population size $\frac{n-n_1}{m_2}$. Whereas conditions (??) and (??) still hold, efficient land-use requires that raising the number of jurisdictions has no effect on the level of utility, formally

$$\left[F_2 - F_{2n} \frac{n - n_1}{m_2} \right] - \rho t - \left[C_2 - C_{2n} \frac{n - n_1}{m_2} \right] = 0, \quad (5)$$

This condition states that the per-capita land rent in excess of the required return on land is equal to the cost of providing public services in excess of congestion costs. In the case of a pure public good, congestion costs are zero and we have the result that the differential land-rent is just equal to the cost of public goods provision, which is the essence of the Henry George Theorem.

If condition (??) is imposed and land size is fixed, the structure of the model is changed. However, the implications of productivity differences for the population size are the same. To see this, note that for a given level of utility, equation (??) is a relationship involving the population size of type-2 jurisdictions and the supply of public services. Taking account of the Samuelson condition (??), for any total population assigned to type-2 jurisdictions $n - n_1$, the central planner can use m_2 to ensure that the population size of the individual jurisdiction is efficient. Finding the efficient population size of jurisdiction 1 reduces to the problem of achieving locational efficiency (??) while satisfying the Samuelson condition. It is obvious that in the symmetric case, where $\gamma = 1$, locational efficiency is ensured if all jurisdictions have the the same size $n_1 = \frac{n-n_1}{m_2}$. In presence of a productivity shock $d\gamma > 0$, locational efficiency requires to increase population size of jurisdiction 1.⁶

Having seen that the jurisdictions differ in population size, regardless of how the supply of land is

⁶The proof of this statement would be similar to the proof of Lemma 1, except that the problem simplifies since land use of each city is treated as fixed.

modeled, let us explore the implications for the efficient provision of public services in the setting with two jurisdictions. To do so, it is sufficient to focus on the Samuelson condition, which provides us with an implicit relationship between the supply of public services z_i , the level of utility u and the population size of the jurisdiction n_i . Because consumption of public goods is non-rival, the size of the jurisdiction affects the cost of public service provision: the marginal cost will rise less than proportionally with population size and consequently $\frac{C_z(z_i, n_i)}{n_i}$ declines. From the Samuelson condition we know that, as a consequence, the ratio of public to private consumption will be higher in the larger jurisdiction. Moreover, with the same level of utility in all jurisdictions, x_i would be smaller in order to compensate for higher z_i . Thus, building on the inverse relationship between the per-capita marginal cost of public good provision and the population size we can derive the following proposition:

Proposition 1 (Cost-Advantage of Cities and Efficient Provision of Public Goods)

If a public good shows some degree of non-rivalry in consumption, in an efficient allocation a larger amount of the public good is provided in the larger jurisdiction.

A formal proof is given in Appendix ??.

3 Size Differences and Public Expenditures

While our analysis shows that, under some relatively weak assumptions, the efficient level of public service provision is higher in the more populous jurisdiction, it is not obvious that public spending would also be higher in per capita terms. If z_i were to stay constant, the per-capita cost $\frac{C(z_i, n_i)}{n_i}$ would decline with population size. However, as shown above, z_i increases with n_i , giving rise

to an opposite effect on public expenditures per-capita. If z_i increases strongly, the latter effect dominates so that the efficient level of public expenditures might rise with a jurisdiction's population size even in per-capita terms. Which effect dominates depends on the Hicksian price elasticity of demand. If the demand for the public good responds rather strongly to a marginal cost-reduction, the per-capita budget will be higher in the larger jurisdiction:⁷

Proposition 2 (City Size and Public Expenditures)

If the cost of providing a public good displays a constant elasticity with regard to the amount provided and if the Hicksian demand for the public good is sufficiently elastic, the efficient level of public expenditure per capita is higher in more populous jurisdictions.

For a formal proof, see Appendix ??.

While it may be difficult to know the value of this elasticity in practice, it is easy to think of examples where the elasticity of substitution is high. Consider, for instance, quality aspects of public service provision such as the purity of water in a city's water supply.⁸ Provided, a certain minimum level of quality is guaranteed, such that it is safe to use the water, the quality might be improved, for example in terms of odor or taste. If people do not care too much about those features, the elasticity of demand might be rather high. In this case, provided the allocation is efficient, our analysis would suggest that these improvements are more likely to be implemented by larger jurisdictions and possibly result in higher per-capita spending. Similar examples along these lines could be found easily also for other functions of government such as public safety, public

⁷This argument might be reinforced in the presence of heterogeneity between households. Consider a case, where two types of households exist, which differ in their preferences for public goods. If larger jurisdictions have a cost advantage in public service provision, it would be efficient to allocate households with a relatively strong preference for public consumption to the larger jurisdiction.

⁸We are grateful to an anonymous referee who offered this example.

transportation, or schooling.

The cost advantage of cities has also been noted by Oates (1988) who argues that it can explain why a greater range of government services is usually provided in large cities.⁹ A crucial condition in this context is the substitutability between private and public goods. If the utility function allows for a complete substitution of public with private consumption, we can determine a level of private consumption \bar{x} consistent with the common level of utility and zero consumption of the public good $\bar{x} : u(\bar{x}, z_i = 0) = u$. The associated marginal rate of substitution $\bar{p} \equiv \frac{u_z(\bar{x}, 0)}{u_x(\bar{x}, 0)}$, can be interpreted as a “reservation price” for the provision of the public service. Only if the marginal cost of public service provision is below the reservation price, the local jurisdiction “participates” in the provision of the service. Thus, we can specify a participation constraint $z_i > 0$, iff $p_i < \bar{p}$. Given non-rivalry in consumption, the marginal cost of public service provision per-capita (p_i) falls with population size and we can find a lower bound for local population \bar{n} consistent with the participation constraint by setting \bar{p} equal to the marginal cost of providing the public good at zero level of provision: $\bar{p} = \frac{C_z(\bar{n}, 0)}{\bar{n}}$.

The relevance of this result is strengthened in the presence of fixed cost. Consider a cost function

$$C(z_i, n_i) = c(z_i, n_i) + kn_i, \text{ for } z_i > 0, \text{ and } c(z_i, n_i) = 0, k = 0 \text{ for } z_i = 0,$$

where kn_i captures the cost of the provision of public services – independent of the level of service provision.¹⁰ With this cost function, the “reservation price” for the provision of the local public

⁹As pointed out by an anonymous referee, the larger variety of public goods in larger jurisdictions, which was empirically observed by Schmandt and Stephens (1960), might induce a process of Tiebout sorting by which the population of larger jurisdictions also displays a greater variety of preferences. This in turn would also imply that the population would demand a more differentiated basket of public goods and services in more populous cities.

¹⁰For analytical convenience we specify the fixed cost k in terms of cost per resident. Introducing a lump-sum amount K , with $C(z_i, n_i) = c(z_i, n_i) + K$, might be more intuitive but complicates the analysis without providing

good will be higher in order to cover fixed cost of k , and we can state the following proposition:

Proposition 3 (Specialization in Private Consumption)

If a public good can be substituted completely by a private good, we can specify a minimum population size below which it is not efficient to provide this good in a given jurisdiction. This minimum population size increases with the level of the fixed cost in the provision of the public good.

A formal proof is given in Appendix ??.

At first sight, it seems to be a rather strong assumption that public services can be substituted entirely by the private good. However, note that complete substitution implies that the *own* provision of public services is zero. In many practical cases, residents of jurisdiction i could still benefit from public services provided in a *neighboring jurisdiction*. While a thorough discussion of benefit spillovers in the context of size differences is beyond the scope of this paper, we may note that the cost advantage of cities in the provision of public services might further contribute to a high level of public service provision in metropolitan areas where households from neighboring jurisdictions also tend to consume the local public services. Of course, in a decentralized setting, a mechanism is required to ensure that the willingness to pay for public services in adjacent jurisdictions indeed results in an expansion of the supply of public services.¹¹

4 Conclusion

This paper has explored the relationship between the population size of a jurisdiction and the demand for local public goods. Under the assumption that public goods exhibit some degree of

important further insights.

¹¹For a discussion of associated incentive problems see the discussion of *superneighborhood goods* by Fujita (1989) and the treatment in Wildasin (2004).

non-rivalry in consumption, larger jurisdictions experience a cost advantage. In an efficient solution, this cost advantage gives rise to a substitution of public for private consumption. If the demand for the public good is elastic or if the public good can be substituted entirely by the private good, the efficient level of public expenditures is higher in larger jurisdictions even in per-capita terms.

Our results support Oates's (1988) concern that empirical estimates of the degree of congestion of local public services are biased. More specifically, our analysis suggests that, without controlling for the level of public services provided, the empirical population size effect not only captures a cost but also a demand effect. While the cost effect works towards lower spending in per-capita terms, the demand effect exerts a positive impact on spending. Which one of these two effects dominates, depends on the properties of the cost-function and on the preferences. Our analysis also shows that smaller jurisdictions will tend to specialize in the consumption of private goods. While we have focused on a single public good, in a setting with multiple public goods also partial specialization may occur, where smaller jurisdiction provide some but not all public services. Hence, the specialization result is in accordance with the notion that higher public spending per-capita in larger jurisdictions also reflects a larger variety of public services.

An important limitation of the above analysis is the focus on a "benevolent" central planner. This assumption is helpful to identify the potential efficiency gains associated with a substitution of public for private consumption in cities. But, of course, our model is silent about whether these gains can be reaped by any specific institutional and political setting.

From a normative perspective, the analysis sheds new light on the fiscal need or expenditure need of cities – concepts which are often used to determine the allocation of unconditional grants among local governments (see Shah, 2007 and Reschovsky, 2007). Several countries allocate funds based

on the assumption that larger jurisdictions have a higher fiscal need in per-capita terms. Other countries use indicators of population density or urbanization in the determination of unconditional grants. As a consequence of these practices, larger cities are often enabled to run larger budgets in per-capita terms compared with smaller jurisdictions. In the public debate these grant allocation rules are often justified by the supposed higher cost of public service provision in cities and, thus, are criticized as a subsidization of cities and their inefficient public sectors. Our analysis shows, quite to the contrary, that an institutional setting allowing cities to run larger budgets in per-capita terms might be efficiency enhancing. Of course, as our analysis shows, this result hinges on preferences and on the properties of the cost function for public service provision. While we have identified crucial factors that support a positive relationship between public spending per capita and city size, it remains an empirical question as to whether this result applies in a specific context.

Though our analysis has highlighted the role of productivity differences related to the private sector, it should also be noted that productivity differences across cities may be associated with the public sector. For instance, some city may implement an innovation in the public sector which reduces the cost of public service provision. Our analysis suggests that this does not imply necessarily that public spending per-capita declines. Since cost-advantages may result in a higher demand, it seems possible that spending per-capita increases. However, to explore the implications further is beyond the scope of this paper and is left for future research.

5 Appendix

5.1 Proof of Lemma 1

Differentiating the optimality conditions (??) to (??) and evaluating the expression at the symmetric equilibrium with $\gamma = 1$, we have

$$\left(F_n + \frac{F_{nt}F_t}{-F_{tt}}\right) d\gamma = \left[-2 \left(F_{nn} \left(1 - \frac{F_{nt}^2}{F_{nn}F_{tt}}\right) - C_{nn}\right) - 2 \left(\frac{\left(-\frac{\partial x}{\partial z} - C_{zn}\right)^2}{n \frac{\partial^2 x}{\partial z^2} + C_{zz}}\right)\right] dn_1. \quad (6)$$

With the bracket on the left-hand side being positive, we note that $\frac{dn_1}{d\gamma} > 0$, if and only if the right-hand side is positive. This condition is equivalent to the second order condition for a utility maximum. More precisely, this condition ensures that the determinant of the bordered Hessian with variables u, n_1, t_1, z_1, z_2 and λ , evaluated at the optimal solution is negative. To see this, consider the determinant:

$$|\mathbf{H}| = \begin{array}{|c|c|c|c|c|c|} \hline -\lambda \left[\frac{\partial^2 x_1}{\partial u^2} n_1 + \frac{\partial^2 x_2}{\partial u^2} n_2 \right] & -\lambda \left[\frac{\partial x_1}{\partial u} - \frac{\partial x_2}{\partial u} \right] & 0 & -\lambda \left[\frac{\partial^2 x_1}{\partial u \partial z_1} n_1 \right] & -\lambda \left[\frac{\partial^2 x_2}{\partial u \partial z_2} n_2 \right] & \begin{array}{l} -\frac{\partial x_1}{\partial u} n_1 \\ -\frac{\partial x_2}{\partial u} n_2 \end{array} \\ \hline -\lambda \left[\frac{\partial x_1}{\partial u} - \frac{\partial x_2}{\partial u} \right] & \begin{array}{l} \lambda [\gamma F_{1nn} + F_{2nn}] \\ -C_{1nn} - C_{2nn} \end{array} & \lambda [\gamma F_{1nt} + F_{2nt}] & -\lambda \left[\frac{\partial x_1}{\partial z_1} + C_{1zn} \right] & +\lambda \left[\frac{\partial x_2}{\partial z_2} + C_{2zn} \right] & 0 \\ \hline 0 & \lambda [\gamma F_{1nt} + F_{2nt}] & \lambda [\gamma F_{1tt} + F_{2tt}] & 0 & 0 & 0 \\ \hline -\lambda \left[\frac{\partial^2 x_1}{\partial u \partial z_1} n_1 \right] & -\lambda \left[\frac{\partial x_1}{\partial z_1} + C_{1zn} \right] & 0 & -\lambda \left[\frac{\partial^2 x_1}{\partial z_1^2} n_1 + C_{1zz} \right] & 0 & 0 \\ \hline -\lambda \left[\frac{\partial^2 x_2}{\partial u \partial z_2} n_2 \right] & +\lambda \left[\frac{\partial x_2}{\partial z_2} + C_{2zn} \right] & 0 & 0 & -\lambda \left[\frac{\partial^2 x_2}{\partial z_2^2} n_2 + C_{2zz} \right] & 0 \\ \hline -\frac{\partial x_1}{\partial u} n_1 - \frac{\partial x_2}{\partial u} n_2 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

Taking into account that $\left(\frac{\partial x_1}{\partial u} n_1 + \frac{\partial x_2}{\partial u} (n - n_1)\right) = \frac{1}{\lambda}$, by expansion, in the symmetric equilibrium with $n_2 = n_1$ the determinant can be simplified to

$$D = -\left(\lambda^2\right) [-2F_{tt}] \left[\frac{\partial^2 x}{\partial z^2} n + C_{zz}\right]^2 \left[-2(F_{nn} - C_{nn}) - 2\left(\frac{F_{nt}F_{nt}}{-F_{tt}}\right) - 2\left(\frac{\left(-\frac{\partial x}{\partial z} - C_{zn}\right)^2}{n\frac{\partial^2 x}{\partial z^2} + C_{zz}}\right)\right],$$

which is negative if the right-hand side of (??) is positive. ■

5.2 Proof of Proposition 1

Let $MRS_i \equiv MRS(x_i, z_i) = \frac{-\partial x(z_i, u)}{\partial z_i}$ be the marginal rate of substitution between public and private goods. Total differentiation of the Samuelson condition for the optimal supply of public goods in jurisdictions i yields the following expression

$$\left[-n_i \frac{\partial MRS_i}{\partial z_i} + C_{izz}\right] dz_i = [MRS_i - C_{izn}] dn_i + n_i \frac{\partial MRS_i}{\partial u} du.$$

Note that this expression holds for both jurisdictions. Since $dn_2 = -dn_1$, the differential impact of an increase in population on the optimal allocation of public goods is unambiguously positive, if the marginal cost of providing public services is non-decreasing $C_{zz} \geq 0$. To see this, evaluate the expression above at the symmetric equilibrium to obtain

$$\frac{dz_1}{dn_1} - \frac{dz_2}{dn_1} = \frac{2\left(\frac{C_z}{n} - C_{zn}\right)}{-n\frac{\partial MRS}{\partial z} + C_{zz}}.$$

Given non-rivalry of public services, $\frac{C_z}{n} > C_{zn}$, the numerator is positive. Also, the denominator is positive, since for a strictly quasi-concave utility function $\frac{\partial MRS_i}{\partial z_i} = \frac{1}{u_{ix}^3} [u_{ix}^2 u_{izz} - 2u_{ix} u_{iz} u_{ixz} + u_{iz}^2 u_{ixx}] <$

0, where we write u_{ix} and u_{iz} for first order differentials and u_{ixx} , u_{ixz} , and u_{izz} for second order differentials. ■

5.3 Proof of Proposition 2

Let $p_i \equiv \frac{\partial C_i}{\partial z_i} \frac{1}{n_i}$ denote the per-capita marginal cost of the provision of public goods or services z_i . $z_i(p_i, u)$ is the Hicksian demand for public services as a function of the marginal cost p_i and the utility level in the economy. The per-capita cost of providing public services is increasing in population size, if

$$\frac{dC(z_i(p_i, u), n_i)}{dn_i} > \frac{C(z_i, n_i)}{n_i}.$$

Inserting the derivative of the cost function yields $\frac{\partial C_i}{\partial z_i} \frac{\partial z}{\partial p_i} \frac{\partial p_i}{\partial n_i} + \frac{\partial C_i}{\partial n_i} > \frac{C_i}{n_i}$. With imperfect rivalry in public consumption, $\frac{\partial p_i}{\partial n_i}$ is negative. Making use of $n_i p_i = \frac{\partial C_i}{\partial z_i}$ we have

$$z_i n_i \underbrace{\left[-\frac{\partial z}{\partial p_i} \frac{p_i}{z_i} \right]}_{\zeta} \left(-\frac{\partial p_i}{\partial n_i} \right) > \frac{C_i}{n_i} - \frac{\partial C_i}{\partial n_i}, \quad (7)$$

where ζ is the Hicksian elasticity of demand. Let $\delta \equiv \frac{\partial C_i}{\partial n_i} \frac{n_i}{C_i}$ be the elasticity of the cost of public service provision with respect to the population size – sometimes referred to as the crowding elasticity. If we define $\pi \equiv \frac{\partial^2 C_i}{\partial z_i \partial n_i} \frac{n_i}{\partial C_i / \partial z_i}$, which is the elasticity of the marginal cost of public service provision with regard to population size, we have $-\frac{\partial p_i}{\partial n_i} = (1 - \pi) \frac{p_i}{n_i}$. Inserting into equation (7), we obtain

$$\zeta (1 - \pi) > \frac{C_i}{p_i z_i n_i} (1 - \delta).$$

If the cost of public service provision displays a constant elasticity with regard to the supply of public services ($\frac{\partial C_i}{\partial z_i} \frac{z_i}{C_i}$), we have $\pi = \delta$, and this condition simplifies to $\zeta > \frac{1}{\frac{\partial C_i}{\partial z_i} \frac{z_i}{C_i}}$. Obviously, this condition is fulfilled for high levels of ζ as stated in the above proposition. ■

5.4 Proof of Proposition 3

Following Cogan's (1981) analysis of labor supply, we study the determinants of the "reservation price" for public service provision by inspecting the minimum expenditures consistent with the economy-wide level of utility

$$e(p_i, u, k) \equiv \min_{x_i, z_i} \{x_i + p_i z_i + I(z_i > 0)k + \mu[u - u(x_i, z_i)]\},$$

where $I(z_i > 0)$ is an indicator function with value one if $z_i > 0$ and zero otherwise. Consider, first, the case without fixed cost, where $k = 0$. We can determine the price $p_i = \bar{p}$ at which the local jurisdiction "specializes" in the consumption of good x such that

$$\bar{x} = e(\bar{p}, u, k = 0).$$

Any decline in the marginal cost of public service provision relative to \bar{p} would support a positive level of local public service provision $z_i > 0$ and private consumption below maximum level $x_i < \bar{x}$. However, if k is positive, a small decline in the marginal cost of public service provision has no effect. Thus, \bar{x} can not be used to determine the "reservation price" for positive levels of k . We can, however, determine a lower level of the marginal cost of public service provision \tilde{p} , which allows the central planner to provide public services at per capita cost of $\tilde{p}\tilde{z} + k$ such that households can

be granted sufficient private consumption in order to sustain the given utility level. Formally,

$$\bar{x} = \tilde{x} + \tilde{p}\tilde{z} + k = e(\tilde{p}, u, k).$$

Differentiating the expenditure function with regard to \tilde{p} and k , and making use of the envelope theorem, we have $\tilde{z}d\tilde{p} + dk = 0$. Rearranging terms, we have

$$\frac{d\tilde{p}}{dk} = -\frac{1}{\tilde{z}} < 0.$$

Thus, an increase in the fixed cost reduces the “reservation price” for public service provision. ■

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